# Possible approaches for the pre-service mathematics teacher's preparation to apply the digital technology in their own teaching

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*The paper presents a study of the development of TPACK of Preservice Mathematics Teachers (PMTs). Two interconnected components of TPACK, the use of digital tools (DT) as instruments for solving problems and for creating digital materials, are dealt with from the perspective of their role in teacher education and mutual relationships. The paper aims to provide examples of how different teaching approaches could develop the knowledge and skills of PMTs in such a way that PMTs will be able to use DT properly in their own teaching.*

*Keywords: TPACK development, PMTs education, digital technology in PMTs preparation, searching and evaluation of existing materials.*

## Introduction

The use of DT in mathematics education at all levels has been a broadly studied topic already for several decades (see e.g. Ruthven, 2007). In 2011, Jančařík and Novotná see one of the reasons for a slow integration of DT into (not only) mathematics education in comparison with the robust development of technological tools for education and the low level of teachers' experience with using it. The rate of this integration increases markedly slowly when compared with the speed of development of technology. Jančařík and Novotná (2011) named as one of the reasons for the situation insufficient teachers' preparation for this integration. Successful use of DT in mathematics classrooms requires that the teacher is able to use it efficiently when solving problems. This is only one of the required competencies. Without additional pedagogical knowledge, e.g. designing lesson plans, selecting suitable materials available (e.g. on the internet), modifying and creating materials intended for the taught group of pupils, evaluating their quality and needed improvements, the use of DT in classrooms remains less efficient.

The research presented in this article is based on a long-lasting collaboration of two groups of researchers/teacher educators at two faculties, the Faculty of Mathematics, Physics, and Informatics of Comenius University in Bratislava and the Faculty of Education of Charles University in Prague. The history of both countries, Slovakia and the Czech Republic, is connected in their past histories, even from the Great Moravia period. The educational systems in the current two independent countries develop separately, but they have many similarities and we found it interesting to look at differences and similarities after almost 30 years of separate history. One of the common topics is an integration of DT into (lower and upper) secondary teacher education. Needing to understand if "insufficient teachers' preparation" is one of the reasons why we analyzed the integration of DT in different preservice secondary mathematics teachers' courses. Teacher knowledge should not only concern the ability to work with DT or solve mathematical tasks. The further important teacher' abilities are proper implementation of DT into the educational process, which includes lesson plan preparation, design and re-design of teaching resources, choosing appropriate and available educational materials, and the design of appropriate assessment and evaluation tools. All these abilities are partial domains of model TPACK that are briefly addressed in the next part of the paper. The collaboration of the two research teams opened new perspectives to handling several questions dealt with by both groups individually. The perspectives stemming from this collaboration can be clustered into two major groups.

The first group (represented here by Case 1) is concerned with the use of DT as a scaffolding and "technical" support when learning mathematics, especially when introducing a new concept and solving problems. This group is linked with improving future teachers' skills in using DT. The second group (represented here by Case 2) is concerned with the use of ready-made digital resources available on the internet and their own production for use in classrooms.

Current research also identified issues such as "limitations of trained staff and the need for practitioners to troubleshoot issues" (Buteau et al., 2010, pp. 58–59). The research question is: What components and forms of the implementation of DT into teacher education should be incorporated into PMTs training and what is the recommended order of their implementation?

## Theoretical framework

The framework for TPACK was introduced by Mishra and Koehler (2006). This framework clarifies the kinds of knowledge required by a teacher in order to ensure the effective integration of technology into their teaching. It can be seen as an enlargement of Shulman's (1986) PCK model with technology as an additional domain.

There are three main competencies in the TPACK model defined by Mishra and Koehler (2006): Technological knowledge (TK), Content knowledge (CK), and Pedagogical knowledge (PK). Technological, pedagogical, and content knowledge (TPACK) is one of four competencies, Technological-content knowledge (TCK), Technological-pedagogical knowledge (TPK), Pedagogical-content knowledge (PCK), and Technological, pedagogical and content knowledge (TPACK), that address how these three main domains interact (see also Fig. 1). TPACK was proposed as the interconnection and intersection of TK, CK, and PK.

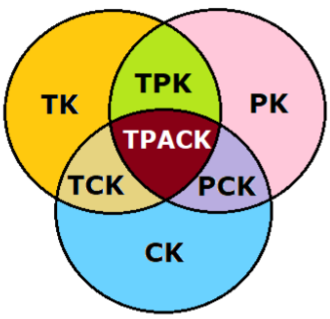


**Figure 1: TPACK framework (based on the image at tpack.org)**

TPACK, according to Koehler et al. (2014, p. 102), “refers to knowledge about the complex relations among technology, pedagogy, and content that enable teachers to develop appropriate and context-specific teaching strategies”. The development of TPACK could be done in three possible ways as identified by Koehler et al. (2014). The first is from PCK to TPACK, the second from TPK to TPACK, and the third, developing PCK and TPACK simultaneously. In our research, we also used the fourth way from TCK to TPACK, which is not mentioned in Koehler et al. (2014). We used it in our previous studies (a summary of those studies is in Slavíčková, 2021). The layering of theoretical frameworks can help us to better understand the studied phenomenon. We used such an approach in Slavíčková (2021) when layering Mishra and Koehler’s framework (Mishra & Koehler, 2006) with Ball’s “egg” framework (Ball et al., 2002).

In this paper, we focus on two from seven components of teachers’ TPACK: the use of DT in mathematical courses (Case 1) and the development of the ability to critically evaluate and modify materials available on the internet (Case 2).

## Methodology

The collaboration of the two teams is ongoing which is why we can present at this point only the results of research anchored in one course at each group level and methodological instruments linked to them. In both cases, a qualitative research design was used.

The Bachelor level mathematics teacher education at both universities focuses on the CK. In Case 1, we focused on PMTs' willingness to use different types of DT in Calculus lessons. The participants were 26 PMTs in the second year of their bachelor study program. We offered them several digital tools like Graphic Calculus, Derive, GeoGebra; they had the freedom to use other devices as well. They were asked to use them not only at home preparation but also in the lessons. The situation in 2020 (and 2021) was more manageable using DT due to the online form of classes.

In the Master’s level mathematics teacher education at both universities, attention is paid to the PMTs’ ability to select and use appropriate digital materials available in a ready-made form and modify them for the conditions of their own teaching. On this level, choosing DT is not guided and it is up to the PMTs judgment to choose the proper one for their lesson preparation. They also learn to design their own original materials. For collecting data about PMTs’ coping with this important activity (Case 2), the course of CLIL (Content and Language Integrated Learning) was selected as representative in our study.

PMT’s worked outside the CLIL course contact lessons. They were asked to choose a material available on the Internet proposed for use in a mathematics CLIL lesson, evaluate it and propose improvements. and justify their proposals. In the research described in Case 2, we worked with 11 students who were in the first year of the Master’s level study. They were used to use DT in the way described in Case 1. Their essays were analyzed and compared. We focused on the selected topics, the nature of criticized issues pointed out and proposed modifications. All analyzed materials were in English. The analyses of essays were accompanied by discussions of the authors of the essays with the researchers.

## Case 1: Development of TCK

As an example of TCK development in PMTs preparation, we choose a Calculus course with the integration of DT at Comenius University in Bratislava. The reason is that Calculus is one of the compulsory courses in PMTs preparation. There were two reasons for the implementation of DT into teaching CK. First, we wanted to demonstrate that DT could help PMTs visualize the situation and construct abstract knowledge. Second, we hypothesized that if students experience teaching new concepts by using DT, they will be more open to using it in their future teaching careers.

### Research description

PMTs used digital environments for modeling different situations in mathematics or real-life connected tasks. We present two examples of such problems

Problem 1: *An account starts with 1 EUR and pays 100 % interest per year. If the interest is credited once, at the end of the year, the value of the account at year-end will be 2 EUR. What happens if the interest is computed and credited more frequently during the year? How can we make 3 EUR at the end of the year?*

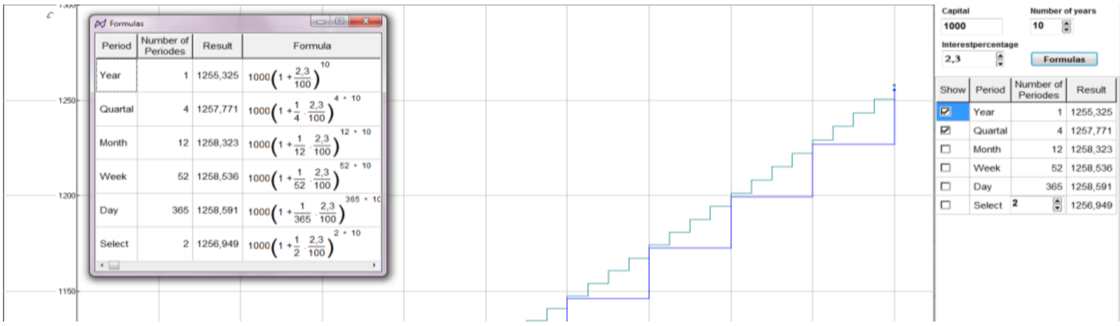


**Figure 2: Using Graphic Calculus for modeling the situation**

As can be observed in Figure 2, a comparison of compound interest calculated every quarter (bigger steps), and compound interest calculated every month (smaller steps) can bring small differences in the final amount of our money at the end of a year. Making these calculations more often does not change the output a lot, the changes are on the positions of thousands or lower. Not all PMTs noticed this phenomenon and the answer, it will never be 3 EUR, was surprising to them. This activity led to the definition of the Euler number in a limit form.

Problem 2: *Using a preferable DT, find out (a) properties of given sequences, (b) categorize them into several categories so every sequence in that category fulfill the condition characteristic for that category, (c) provide example of at least 2 sequences for each determined category.*

$$\left\{\frac{1}{n}\right\}, \left\{1-\frac{5}{2n}\right\}, \left\{3+\frac{2}{5n}\right\}, \left\{\frac{n+5}{n^2+7}\right\}, \{(-1)^n\}, \{\cos(\pi n)\}, \{7.(-1)^n\}, \left\{(-1)^n\frac{n+5}{n^2+7}\right\}, \{2^n\},$$
$$\{n+5\}, \{9n+7\}, \left\{\frac{n^2+5}{n+7}\right\}, \{7-n^2\}$$

*Will your categories work for other sequences as well? Justify your answer.*

When working with this problem, we followed the spiral scheme Manipulating–Getting-a-sense-of–Articulating–Manipulating–… described by Mason and De Geest (2010). We merged some types by articulating and further manipulating the original 7 categories of sequences (increasing, decreasing, lower bound, upper bound, bounded, no bounds, periodic). We started activities related to the intuitive understanding of a limit of a sequence. The role of DT was crucial here since the possibility of visualization, zoom, and change the interval for making observation and getting a sense of noticed phenomenon, helped students predict the situation for a "larger *n*".

Introductory problems or tasks were introduced also for the limit of a function, derivatives of a function, the definition of Riemann integral, etc. Different GeoGebra applets and software mentioned above were used during the lessons for the discussions and for home preparation.

**Results**

There are two main results we observed during the teaching period. Firstly, PMTs who actively used DT were having fruitful discussions during the lessons. We observed that mostly those who have a high level of TK were in that group of PMTs. Additionally, nowadays this group of PMTs in their Master's level (2 years after the intervention) is more open to integrating DT into their lesson plans. Unfortunately, PMTs mostly focused on repeating existing knowledge, and most of the tasks they re-design were strongly procedural.

PMTs proved high flexibility in the 2nd semester. The guidance from the teacher was not as significant as in the 1st semester of Calculus. PMTs started looking for different types of software that would help them to manage the abstract concepts they had to learn.

**Discussion of Case 1 results**

In the curriculum, PMTs in their 2nd year of study at the university had only one subject focusing on the development of the three main domains identified by Mishra and Koehler (2006): TK, CK, PK. The course of calculus was the first in which TCK started to be developed. From the results, we can conclude that PMTs development of TCK was in general sufficient. Following our previous results (Slavíčková, 2021), PMTs with lover DT skills were less willing to use DT. This phenomenon changed in 2020 when the COVID-19 situation pushed us all to use them in everyday life.

The reason for procedural-oriented outcomes of PMTs could be our colleagues' teaching when they stress procedures or algorithms for solving typical tasks. PMTs could conclude that mathematics aims to manage plenty of procedures. Single intervention is not sufficient here, and more collaborative work is needed. Therefore, cooperation with other colleagues preparing lessons focusing on using DT, in general, is crucial.

## Case 2: Further development of TPACK

As an example of a suitable activity in a course, for future mathematics teachers aimed to develop their ability to choose and modify existing materials available on the internet, we choose the course of CLIL (Content and Language Integrated Learning) at the Faculty of Education of Charles University.

Coyle, Hood, and Marsh (2010, p. 3) characterize CLIL as follows: “CLIL is an educational approach in which varies language-supportive methodologies are used which lead to a dual-focused form of instruction where attention is given both to the language and the content.” CLIL teaching units have two educational goals of the same importance (language and content). Teaching and materials have to pay attention to both. Both educational goals are interconnected even if the teacher or students focus on one of them. The dual-focused nature of CLIL offers a rich resource of ready-made digital teaching materials. They can focus on the non-linguistic subject, language or, in the ideal case, on both at the same time. An example of such dual-focused material is presented e.g. in (Hofmannová & Novotná, 2007).

*Evaluations of materials*

Students focused on both, mathematics and language. From the language perspective, they paid attention mostly to the used vocabulary and mathematical terminology, and suitability of the used language level from the perspective of the target group of pupils. They proposed reformulations of parts where they considered the language too complicated (including the length of paragraphs etc.). In one case they evaluated the used language as outdated and recommended to change it to the contemporary forms. They recommended modifications of the problem settings aiming to have the texts more similar to the Czech environment.

They also mentioned the differences in notation (e.g., in writing dates or mathematical notations). In this case, they did not propose to use the Czech way but to practice the English one continuously so that pupils get used to using it.

They underlined the suitability of the use of problems with more than one correct result and solving procedure. As to the tasks, they evaluated dividing complex tasks into simpler ones as more suitable. They also paid attention to the quality and correctness of figures and the level of difficulty of tasks.

They mentioned also the layout of the material, mainly the comprehensibility of the text and aims of the activities, and proposed improvements. In cases where the answers to the tasks were pre-prepared, the students proposed deleting it and letting pupils formulate the answers themselves.

The students asked for complex materials where the teacher can find all information needed as help for the teacher. One student worked with a whole book and recommended it as an inspiration for the teacher’s preparation of lesson plans.

**Discussion of Case 2 results**

The students who participated in the CLIL course did not have the courses of didactics of mathematics completed. Nevertheless, the majority of them were able to choose materials suitable for use in their CLIL classrooms and propose meaningful modifications. They presented their pieces of work in CLIL course lessons and discussed deeply the quality and usefulness of materials. They used their experiences from the courses of Didactics of mathematics. Their experiences from the use of DT in mathematics courses helped them to overcome obstacles based on the implementation of DT.

The participating students were aware of the importance of similar activities. Let us cite from one student's comment: "*Given how much internet is infested by materials of bad quality, it was nearly impossible to find materials meeting my requirements …*"

## Conclusion

The two analyses allowed us to estimate the role of PMTs' mastering the use of various DT as instruments for learning mathematics and solving problems themselves on the one hand and working with digital materials during their teaching at schools.

Answering out our research question, *what components and forms of the implementation of DT into teacher education should be incorporated into PMTs training and what is the recommended order of their implementation*, we identified several aspects which should be considered. Firstly, PMTs should start with the development of TCK and TPK sooner than on the Master level. Secondly, using different topic areas and making connections between the main domains of TPACK can help PMTs gain better insight into the issues of implementation of DT into their teaching. Thirdly, closer communication among PMTs' educators is needed, especially those focusing on three main domains of the TPACK model, in other words, educators responsible for mathematics preparation, technology preparation, and pedagogy preparation. All of them should communicate with each other and with a specialist in mathematics education to be concise and help develop TPACK. Fourthly, our observations and analyses of materials produced by PMTs indicate that PMTs' level of TPACK is sufficient for their successful implementation of DT in their classrooms once they enter the practice. They are able to work with ready-made materials and modify them in a creative way. When doing so, they combine competencies gained in the use of DT in their own learning of mathematics and solving problems, as well as general and subject didactics.

Based on our research, recommendations concerning implementing different activities connected to using DT in PMTs preparation can be drawn. It turns out that starting with TCK development is a good starting point (Slavíčková, 2021). Then we could continue with pedagogy-oriented activities (like in Case 2), re-designing the given (or found) materials to create an own material by using DT. When PCK is developed, it can be quite late to start with TPACK. As Mishra and Koehler (2006) identified, once the teachers are familiar with processes and lesson design without using DT, it is difficult to change their mindset.

The number of participating PMTs and courses was small, and the presented results cannot be generalized. Still, they indicate the importance of both components of including DT in (not only) mathematics teacher education. This study opened new questions, e.g., how well-equipped are our PMTs for their real teaching? How can we measure that? What are the indicators of a well-prepared teacher? This will be the focus of our subsequent research.

Cooperation between our universities continues. We are preparing further interventions and comparisons of our PMTs' results e.g. when creating lesson plans or in their flexibility in adapting their teaching to different environments (mainly by using digital tools).

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