Differences in the comprehension of the limit concept and desired “connected knowing” in calculus between prospective mathematics teachers and managerial mathematicians

Mária Slavíčková[1[0000-0002-3357-2502]] and Michaela Vargová[1]

[1] Faculty of Mathematics, Physics and Informatics, Comenius University in Bratislava, Slovakia;
slavickova@fmph.unba.sk

1 Introduction

Calculus (at least on the basic level) is intertwined through all STEM-oriented university studies. It is a critical milestone in various transition processes from secondary school to university mathematics. For many students it is “the necessary evil” to pass through, and many of them struggle in their courses. One reason of this struggling could be the huge difficulty of linking between the knowledge of mathematics learned at university and the knowledge acquired in secondary school. This gap between mathematical levels and institutional cultures can lead to several study problems of freshmen. As Pinkernell [1] summarizes, students meet different level of rigour in communication or reasoning, and institutional differences, e.g. concerning the didactic of teaching and learning mathematics. The other problem could be that situation at some study programmes is almost the same as one of the interviewees said in research made by Bosch et al [2], “…the exercises is a list that comes from father to son. It’s the same list that has been there for the past 10 years. […] the key for 60% or 70 % of the students to pass is to do an exam that is not essentially different from previous one”. Critical reason could be also that many students are only passive listener and users of calculus. The algorithmic characteristic of the tasks solved in the lessons and tests can lead to passing through exams without deeper understanding on subject matter. In terminology of Boaler and Andrew-Larson [3], most of our students have “received knowing”, which means, they believe that doing mathematics means to memorize and quickly recall information needed.

2 Methods and chosen tasks

We observed the lessons and tested two groups of students at Faculty of Mathematics, Physics and Informatics, Comenius University in Bratislava (Slovakia) on comprehension of the definition of sequence limit (how can small changes, e.g., order in quantifiers, in definition of specific concept influence the meaning of the definition). The first group comprised 24 pre-service mathematics teachers (PMTs), the second group comprised 28 managerial mathematics students (MNGs). Based on our previous research, [4], freshmen are struggling with basic properties of functions (mostly goniometric and logarithmic), have no experiences with formal mathematical notation and therefore are not able to follow the lectures. Moreover, they have problems with logical structure of statements containing several quantifiers and lack experience with rigor in reasoning. Due to COVID-19 we had to shift into online environment. We were looking for the answer for our research question: *How the ways of reasoning in the groups of PMTs and MNGs differ when teaching/learning in online environment?*

In students' solutions, we looked at the type of argument elicited (in terms of Bieda et al [5]). The test asked students to provide an argument in any form (empirical argument, provide counterexample, formal proof, etc.) using different representations (e.g. graphical, symbolical, verbal, etc.).

The tasks we gave them were similar to the examples below:

1. Let real number L be the limit of sequence $\{a_n\}_{n=1}^{\infty}$. Can the number L be one of the terms of this sequence?
2. Considering the following statements decide how is the correct definition of limit of sequence violated. For every provided statement show an example of a sequence types, which will be convergent within it.
   a. Real number L is called the limit of a sequence $\{a_n\}_{n=1}^{\infty}$if for every $\varepsilon > 0$ there exist an $n_0 \in \mathbb{N}$ such that $|a_n - L| < \varepsilon$ for infinitely many $n > n_0$.
   b. Real number L is called the limit of a sequence $\{a_n\}_{n=1}^{\infty}$ if for every $n_0 \in \mathbb{N}$ there exist an $\varepsilon > 0$ such that $|a_n - L| < \varepsilon$ for all $n > n_0$.

We discussed our results with relevant literature concerning teaching and purposes of calculus at universities (like [6, 7]).

3 Results

Even though two studied groups of students had different backgrounds and mathematical training at the university, there are no significant qualitative differences between these groups when answering our questions. On the other hand, we observed higher effort to reason and prove the answers in the group of MNGs.

The most common argument in both groups was by providing a counterexample. Even though students during the semester encountered several different representations of sequences and their limit (graphical, algebraical, numerical, topological), the most popular way of solving the tasks was graphical, by using epsilon stripes.

Several misconceptions were identified. The most common were epistemological obstacles, when students applied properties of finite sets to the infinite ones (as described in [8]), problems caused by fundamental linguistic flaws in the standard presentation of limit (as identified in [9]) and misunderstanding of the quantifier logic in mathematical statements.

References

1. Pinkernell, G.: Conceptualising knowledge of mathematical concepts or procedures for diagnostic and supporting measures at university entry level. In: Eleventh Congress of the European Society for Research in Mathematics Education, Feb 2019. Utrecht University, Utrecht (hal-02422666)
2. Bosch, M., Hausberger, T., Hochmuth, R., Wilsløw, C.: External Didactic Transposition in Undergraduate Mathematics. In: Eleventh Congress of the European Society for Research in Mathematics Education, Feb 2019. Utrecht University, Utrecht. (hal-02422580)
3. Boaler, J., Andrews-Larson, C.: Identity, agency, and knowing in mathematics worlds. In. J. Boaler (ed.) Multiple perspectives on mathematics teaching and learning, pp. 171-200. Ablex, Westport (2000)
4. Vargová, M., Slavíčková, M.: Students' misconceptions in definition of a limit of sequence. In. 19th Conference on Applied Mathematics, pp. 1089-1097. Spektrum STU, Bratislava (2020).
5. Bieda, K. N., Ji, X., Drwencke, J., Picard, A.: Reasoning-and-proving opportunities in elementary mathematics textbooks. International Journal of Educational Research 64, 71–80, (2014)
6. Herrera, G., Rivera, A. F., Aguirre-de la Luz, K.: Calculus students' difficulties with logical reasoning. In: Eleventh Congress of the European Society for Research in Mathematics Education, Feb 2019. Utrecht University, Utrecht. (hal-02422637)
7. Voigt, M., Rasmussen, C., Martinez, A.: Calculus variations as figured worlds for math indentity development. In: Eleventh Congress of the European Society for Research in Mathematics Education, Feb 2019. Utrecht University, Utrecht. (hal-02459946)
8. Davis, R. B., Vinner, S.: The notion of limit: Some seemingly unavoidable misconception stages. The Journal of Mathematical Behavior, 5(3), 281–303. (1986)
9. Swenton, F. J.: Limits and the system of near-numbers. International Journal of Mathematical Education in Science and Technology, 37(6), 643-663, (2006)