TEACHERS' REASONING IN DIFFERENT TOPICS OF SCHOOL MATHEMATICS

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Abstract

Nowadays is sometimes called the post-factual era, when the distinction between truth and false is being blurred. The need to verify information, to think critically about it and to argue correctly is therefore greater than ever. One of the aims of mathematics is to develop these skills. In an international survey, we investigated which areas of mathematics teachers include reasoning and proving tasks the most and the least in. We were also interested in what the concepts of reasoning and proof mean to them in mathematics teaching. A questionnaire was developed and used in the involved countries which were Slovakia, the Czech Republic, Italy, Norway and Turkey. The teachers in our sample use reasoning and proving tasks mainly in geometry or algebra as expected while the least in the area of data and function. In accordance with literature there are many misconceptions in teachers´ perception of a role or nature of reasoning and proving in school mathematics. For instance there was a common opinion among the teachers that activities based around reasoning and proving are only for gifted students.

Keywords: Reasoning, proving, comparison, topics, international, curriculum.

1 INTRODUCTION

One of the goals of teaching mathematics is to increase students' competence in reasoning. Pupils learn this skill mainly implicitly, and throughout all their years of education. The ability to argue correctly and think critically is not only applied in further mathematics studies and other scientific disciplines, but also in everyday life. With the amount of information we receive nowadays, the ability to verify its veracity and assess its meaning is much more necessary than it has ever been before. Although some topics in school mathematics offer tasks that are firmly linked to argumentation, in others the argumentative potential of the tasks needs to be discovered and then used appropriately by the teacher. What are the topics where teachers most often include argumentation and proof tasks in primary and secondary school? Is the list of these topics culture-dependent, depending on national curriculum or are these universal and these topics are the students' gateway to argumentation and proof for a different reason? Do teachers see proof and argumentation as an important part of the curriculum and what do these concepts mean to them in the classroom?

2 METHODOLOGY

To find answers to these and many other questions about argumentation and reasoning in primary and secondary schools, we conducted a questionnaire survey in the countries participating in the Horizon 2020 MaTeK project (Enhancement of Research Excellence in Mathematics Teacher Knowledge). There are five countries involved in the research: Slovakia, the Czech Republic, Italy, Norway and Turkey. The questionnaire was designed in English and involved all participating countries so that the questions were meaningful in each school system and the results could be compared. The questionnaire was then translated into each language, with multiple cross-checks. Before pilot study was conducted, the questionnaire was tested and adjusted accordingly to increase its validity and reliability. The questionnaire included, among other things, a question on the range of topics in which teachers include argumentation and proofs. Respondents also indicated how often they included such tasks in a given topic.

2.1 Question Structure

The questionnaire consisted of 29 items. Six questions were dedicated to teacher's use of resources in different situations, two questions asked about Covid-19 pandemic impact on their use of resources, while seven questions were connected to reasoning and proving. Other questions were demographic. Questionnaire consisted of mainly closed but also open ended questions. In this paper, we focus on three particular items related to argumentation during the lesson, one of which was open-ended and explored teachers' conceptions and ideas about argumentation. Teachers also provided an example of argumentation or proof that they use in the classroom.

The closed pair of questions concerned the areas in which teachers included argumentation or proof. Because curricula vary from country to country, it was necessary to find areas of intersection that are taught in all countries. Therefore, in the end we stuck to a general division into the following five areas: *Geometry*, *Arithmetic*, *Algebra*, *Data* (Combinatorics, Probability & Statistics) and *Functions*.

After the respondent indicated which areas were relevant to his/her teaching, he/she selected the frequency of including reasoning or proving activities in his/her mathematics classes for each of the areas he/she teaches. The frequency was chosen by the teacher on a four-point Likert scale with options of *not at all*, *sometimes*, *usually* and *always*.

2.2 Pilot Study

In the pilot phase, the questionnaire was disseminated via social media groups for mathematics teachers. The questionnaire was open to receive responses for two weeks. During this time, we received responses from 15 Slovak, 21 Czech, 32 Italian, 20 Norwegian and 22 Turkish teachers (see Figure 1) with different teaching experiences (see Table 1).

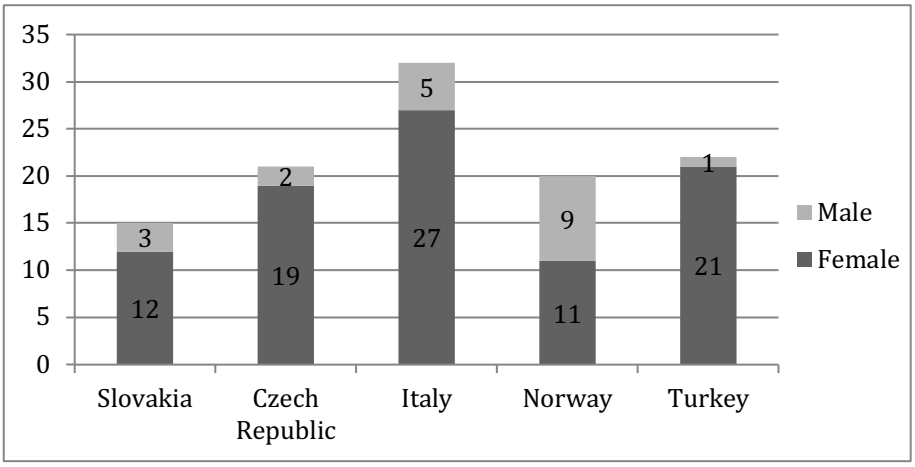

Figure 1. Number of participants and their gender.

It is generally known that there are more female in-service teachers than male. This phenomenon is observable also from our pilot study (Figure 1). The ratio of female and male in-service mathematics teachers can be different in the population.

Table 1. Length of respondent's praxis in years.

Years of teaching	Slovakia	Czech Republic	Italy	Norway	Turkey
1st	0	3	1	2	0
2-5	3	10	4	8	9
6-10	4	2	5	3	6
11-15	2	0	1	4	2
16-20	1	0	5	0	5
More than 20	5	5	16	3	0

The frequency of use of reasoning and argumentation tasks was converted to a numerical value from 0 to 3, where 0 means never and 3 means always. A weighted average was then calculated so that the values could be compared between mathematics domains and countries. These averages are shown in Table 2. The table also shows the highest averages in bold, which represent the area in which the interviewed teachers from a given country most frequently included argumentation and proof tasks. In contrast, the areas with the lowest averages are underlined.

3 RESULTS

In all five countries, teachers voted for the area with the highest concentration of reasoning tasks and proofs in one of two areas, *Geometry* or *Algebra*, which is in line with expectations, since historically the first proofs appeared in the area of geometry, the teaching of which is based from the Euclidean tradition (300 BC) [1]. On the other hand, the least frequently interviewed teachers justify and prove with their students in the areas of *Data* (this area ranks last in three countries - Slovakia, Czech Republic and Italy) and *Functions* (Norway and Turkey). The reason for the lower representation of reasoning and proving in this area may be partly due to the fact that the area is not explicitly as visual, but also due to its significantly lower representation in the curriculum compared to, for example, *Geometry*. Although one could argue similarly in the area of algebra, unlike statistics, in practice it very often converts abstract relations into geometric relations, and thus often argumentation in algebra means converting to argumentation in geometry, where students find reasoning from the answers given in the open-ended questionnaire items most natural.

Table 2. Frequency of inclusion of justification in each topic with extremes highlighted.

	Geometry	***Arithmetic***	***Algebra***	***Data***	***Functions***
Slovakia	1,60	1,67	**1,93**	1,43	1,73
Czech Republic	**1,85**	1,76	1,63	*1,15*	1,23
Italy	**2,78**	2,56	2,52	2,41	2,66
Norway	1,80	1,85	**1,90**	1,80	*1,32*
Turkey	**2,14**	2,10	1,95	1,86	1,39

The answers that teachers gave as examples of argumentation were in most cases also geometric (Pythagorean theorem, Euclidean theorems...), rarely from algebra (calculating the roots of a quadratic equation).

Teachers in the study sample are aware of the importance of reasoning and proving in mathematics teaching: "...mathematics is not a religion it requires explanations, justifications (More than 20 years of teaching experience)." However, there were also dissenting views that expressed a rather dismissive attitude towards argumentation and proof: "...for me it is only delay in classes, pupils are not willing to think or listen (More than 20 years of teaching experience)." Other teachers, on the other hand, believe that the area is more suitable only for gifted pupils: "For the more gifted pupils it is an insight into mathematics (2-5 years of teaching experience)." Knuth [2] also mentions the same attitude of teachers towards reasoning and proving. He also talks about the strong need for teachers to check the correctness of a proven proposition in an empirical way. Teachers in our sample also mention empirical proving and some consider it an adequate way of justifying: "Verifying that a formula is valid, with a concrete example (2-5 years of teaching experience)."

Teachers' ideas about the form of argumentation in the classroom also vary widely. Some consider it important that they themselves give the argument for use or validity: "Explain to the pupils also the essence of why it is used that way, not just stating thus it is true (6-10 years of teaching experience)." Others, in line with [3], regard argumentation as a collective activity: "We don't just present ready-made arguments to pupils, we try to derive them together, to prove them (11-15 years of teaching experience)."

4 DISCUSSION

Despite small sample size, the views and conceptions of teachers on reasoning & proving in school mathematics were in accordance with literature and other researches (for example study with pre-service teachers in [4]). Views on argumentation and justification vary among teachers in practice and cover the whole range from no argumentation to rigorous deductive proof.

We found surprising result in the pilot data: although we expected similar results in the Slovakia and the Czech Republic due to their long common history (see, e.g., [5]), our current survey does not confirm this (except in the area where teachers reason and prove the least: *Data*). However, this may also be due to the small sample size from the pilot testing. Other surprising result was already mentioned least use of reasoning and proof in Data, since this is the topic in which we can see plenty of opportunities to reason (e.g., [6]; [7]). Norway and Turkey are the least likely to include reasoning and justification in the topic unit *Functions*.

5 CONCLUSIONS

Several trends can be observed from the preliminary results. The data collected from Italian teachers show that teachers include argumentation and evidence equally regardless of the topic. This may be due to both the larger time allocation allocated to mathematics lessons and the type of textbooks. These tend to be more in the nature of college textbooks. We are currently processing the results from the main testing, in which more teachers participated. A deeper insight into the issues would be gained from interviews, which we also plan to conduct with teachers in the participating countries. In September 2022 the main data collection was completed. It will be of interest to compare and observe to which extent it will validate our pilot results. Further interviews around these questions with teacher will also follow.

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