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Introduction

A lot of questions which are at the core of optimization problems are already asked by both primary (age 6 – 15) and secondary (age 15 – 19) school pupils. Nevertheless, it is only quite late (if at all) that the answers to these questions are provided. This is because in the Czech curriculum optimization is taught mainly after derivatives. This sudden appearance of new type of problem causes many obstacles in pupils' understanding. To avoid this, a series of optimization problems and methods, usable at either primary or secondary school, was collected from the history of optimization. One of the methods identified is based around using physical representations.

A series of semi-structured interviews based around this visual method was conducted. The main focus of interviews was to analyze the way pupils react to uncommon and novel methods of problem solving. I sought to find out if physical representations could be suitable for classrooms and to find what advantages and disadvantages such demonstrations might bring for both pupil and teacher.

Theoretical background

Many problems from the field of optimization were solved using parallels with the physical world. For instance, *Heron's problem* was solved by observing the way light beams travel (Rojo & Bloch, 2018) or *Snell's law*, from physics, was used to solve the *brachistochrone problem* by Bernoulli (Tikhomirov, 1986). This approach is not as common today, as it lacks rigor and generality (Polya, 1954). Levi (2009) states that mathematics and physics are so intertwined that one without the other is deprived. Levi (2009) also considers mechanics to be geometry focused on touch and movement. These two elements add extra benefits which geometry otherwise lacks. Using physical experiments allows pupils to perceive the problem with other senses, which leads to multisensory learning.

Methods

For the research, a series of interviews was conducted. Interviews were in form of a dialog between participant and researcher (myself), as I wanted to observe the way the pupil worked with manipulatives. The length of interviews varied from 30 minutes to one hour depending on the pupil's ideas, focus, and will to go on. The research sample consisted of 8 primary and 9 secondary school pupils. Pupils from primary school were selected based upon their positive attitude towards the subject and their skills in mathematics and physics. Pupils from secondary school were volunteers willing to participate in the research. Tasks selected for the interviews were picked based upon two criteria. First, the problems had to be clear enough for both primary and secondary school pupils to understand, and second, the experiments via which the problems were solved had to be real-life performable (not merely thought experiments). The first problem discussed was *Steiner's* *problem*. Pupils were to use the soap films to find the *Toricelli point* and its properties (Courant & Robbins, 1996). A special contraption consisting of two transparent rectangular plates connected to each other by three columns was used. This contraption was submerged into soap water and taken out. Between the plates and the columns, soap films were formed. When looked at from above, we saw a perfect 2D representation of the shortest path with the *Toricelli point* as an intersection of three soap films. The second approach used three weights of the same weight and three pieces of string of arbitrary length. The strings were tied together and the other end was attached to the weights. Three pulleys were needed as well. After we hung the contraption on the system of the pulleys (which formed a triangle) the weights would eventually find a state of equilibrium and would not move anymore. The position of the knot which tied all three strings together represented the *Toricelli point* (Levi, 2009).

Results

It was difficult for pupils to work with manipulatives on their own. They often gave up quickly and had to be guided through the experiment. Their guesses were based solely on visual impressions that the point (intersection of strings/films) is a center of gravity, inscribed or circumscribed circle etc. Pupils mostly abstracted from the physical world and looked at the results as if they were only 2D representations on paper. They saw films and strings as lines and ignored weights, gravity or energy necessary for understanding. After pupils understood the meaning behind the experiments, they were fascinated by the fact that it works. That was an important moment, as the pupils themselves were motivated to look for explanation. Both experiments awoke curiosity in pupils as they asked “why” and “how”. After initial struggle, pupils considered the explanations to be easy and playful. From that I assume this kind of experimentation could be powerful in the classroom as it makes the problems accessible for younger children and motivates them to learn. Several misconceptions in the children’s understanding were revealed as well that might otherwise remain hidden. This suggests the experiments could serve as a diagnostic tool for teachers.

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