González, G., & Skultety, L. (2018). Teacher learning in a combined professional development intervention. *Teaching and Teacher Education, 71*, 341–354.

Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shanan, E., & Williamson, P. (2009). Teaching practice: a cross-professional perspective. *Teachers College Record, 111*, 2055–2100.

Herbst, P., Chazan, D., Chen, C., Chieu, V.M., & Weiss, M. (2011). Using comics-based representations of teaching, and technology, to bring practice to teacher education courses. *ZDM Mathematics Education, 43*(1), 91–103.

Herbst, P., & Kosko, K.W. (2013). Using representations of practice to elicit mathematics teachers' tacit knowledge of practice: a comparison of responses to animations and videos. *Journal of Mathematics Teacher Education, 17*(6), 515–537.

Nohda, N. (2000). Teaching by open-approach method in Japanese mathematics classroom. In T. Nakahara & M. Koyama (Eds.) *Proceedings of PME 24 (Vol. 1)* (39–53). Hiroshima University.

Samková, L. (2019). Preparing future teachers for formative assessment: the case of Concept Cartoons. In J. Novotná & H. Moraová (Eds.), *Proceedings of SEMT '19* (pp. 372–382). Univerzita Karlova, Pedagogická fakulta.

Samková, L. (2020). The typology of arithmetical Concept Cartoons. *South Bohemia Mathematical Letters, 28*, 28–36.

Samková, L. (2021). Preparing future teachers for formative assessment: the combination of a video recording and a Concept Cartoon. In J. Novotná & H. Moraová (Eds.), *Proceedings of SEMT '21* (pp. 367–377). Univerzita Karlova, Pedagogická fakulta.

Skilling, K., & Stylianides, G.J. (2020). Using vignettes in educational research: a framework for vignette construction. *International Journal of Research and Method in Education, 43*(5), 541–556.

Wieman, R., & Webel, C. (2019). Patterns linking interpreting and deciding how to respond during the launch of a lesson: noticing from an integrated perspective. *Mathematics Teacher Education and Development, 21*(1), 28–50.

SUPPORT OF ARGUMENTATION AND REASONING WITH 11-12-YEAR-OLD PUPILS

Mária Slavíčková, ☜ ***Jarmila Novotná, Jakub Michal*** ☜☜ ***and Peter Vankúš*** ☜

Abstract

The workshop will focus on argumentation and reasoning. Arguments suitable for smooth transition between primary and lower secondary levels will be presented, exemplified, analysed, and discussed.

☜ Comenius University in Bratislava, Slovakia; e-mail: slavickova@fmph.uniba.sk; peter.vankus@fmph.uniba.sk

☜☜ Charles University, Czech Republic; e-mail: jarmila.novotna@pedf.cuni.cz; cjakub@email.cz

Keywords: argumentation, reasoning, mathematics, textbooks, problems

Theoretical background

The significance of textbooks in the educational system is widely acknowledged, and their content plays a crucial role in student learning (Haggarty & Pepin, 2002). Various scholars in the field of mathematics education have focused on identifying the different forms of reasoning that are communicated through textbook problems (e.g., Silverman & Even, 2015). Stylianides (2009) developed a framework that can serve as an analysis tool for textbook examination and a teaching aid in professional development sessions for teachers. Similarly, Sevinc et al. (2022) presented an integrated framework for examining modes of reasoning in mathematics textbooks, which is most relevant to our workshop (see Table 1).

Ways of reasoning	**Short characteristic**
1) Appeal to authority	No reasoning
2) Simple (1-step) deduction	Using one premise
3) Mathematizing	Decontextualization
4) Reasoning by analogy	Using several cases
5) Reasoning with empirical arguments/specific cases • Making claims and generalizing • Justification of claim	Generalization from specific cases
6) Developing conclusions/justifying/ rejecting through deductive reasoning • Generic example • Counterexample • Systematic enumeration • Other	Conclusion derived from the given information
7) Other	e.g., abductive

Table 1: Categorization of R&P tasks based on the framework Sevinc et al. (2022, p. 2085)

To focus our work, we limited ourselves to specific areas of mathematics presented in two textbooks for 5th grade students (aged 11-12). After reviewing available textbooks in Slovakia and the Czech Republic, we chose two: Novotná et al., 1996 (referred to as Textbook A) and Šedivý et al., 2001 (referred to as Textbook B). We selected these books for their unique approaches to presenting

new ideas and their focus on discovery and problem-solving.

Textbook A approaches mathematics from a practical perspective, encouraging students to solve problems from various fields and everyday situations. This textbook emphasizes experimentation, modelling, data collection and analysis, and critical thinking to help students find optimal solutions through various methods. The book encourages an active approach to learning, allowing students to discover new concepts on their own and draw their own conclusions

On the other hand, Textbook B is widely used in lower secondary schools and is popular among mathematics teachers. This textbook introduces new concepts through short activities and familiarizes students with new terminology and procedures. While the wording in this book is not explicitly focused on reasoning, many concepts are implicitly presented through the questions posed to students

Description of the planned sessions

Our workshop is designed to provide participants with a comprehensive understanding of three distinct perspectives, each discussed in the separate section. In the first two slots we will focus on two interconnected perspectives: (1) identifying who takes an active role in educational situations presented in textbooks, (2) the types of reasoning and proof (R&P) tasks that are presented in textbooks.

The first slot will be focused on the algebra and arithmetic, the second one on the geometry. By doing so, participants gain a better understanding of the various stakeholders involved in the educational process and can develop strategies to address their unique needs and learn about different R&P task categories and how to sort them using the framework presented in Table 1.

The final slot of our workshop will be dedicated to the re-design of existing mathematical tasks to support their potential for the development of R&P skills. Through this exercise, participants will learn how to identify key areas for improvement in existing tasks and develop strategies to modify them. This is an important skill for educators who want to ensure that their students are prepared to tackle real-world challenges through the development of R&P skills.

Throughout the workshop, participants will have the opportunity to engage in hands-on activities and collaborate with peers to develop a deeper understanding of the material. Our goal is to create a supportive learning environment where participants can feel comfortable asking questions and sharing their ideas.

We believe that the knowledge and skills gained from this workshop will be highly beneficial to educators, curriculum developers, and anyone interested in improving the quality of educational materials. By focusing on the three perspectives outlined above, participants will be better equipped to create engaging and effective educational experiences that promote the development of R&P skills.

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References

Haggarty, L., & Pepin, B. (2002) An investigation of mathematics textbooks and their use in English, French and German classrooms: Who gets an opportunity to learn what? *British Educational Research Journal*, *28*, 567–590.

Novotná, J., Kubínová, M., Sýkora, & Sinková, M. (1996). *Matematika s Betkou 1* [Mathematics with Betka 1]. Scientia.

Sevinc, S., Kohanová, I., Isiksal-Bostan, M., Kubáček, Z., Isler-Baykal, I., Lada, M., Cakiroglu, E., & Di Paola, B. (2022). Developing an integrated framework for analyzing ways of reasoning in mathematics. *ICERI2022 Proceedings* (pp. 2082–2089). https://dx.doi.org/10.21125/iceri.2022.0529

Silverman, B., & Even, R. (2015). Textbook explanations: Modes of reasoning in 7th grade Israeli mathematics textbooks. In K. Krainer, & N. Vondrová (Eds.), *Proceedings of the Ninth Conference of the European Society for Research in Mathematics Education* (CERME9) (pp. 205–212). Univerzita Karlova.

Stylianides, G. J. (2009). Reasoning-and-Proving in School Mathematics Textbooks. *Mathematical Thinking and Learning*, *11*(4), 258–288. https://doi.org/10.1080/10986060903253954

Šedivý, O., Čeretková, S., Malperová, M., & Bálint, Ľ. (2001). *Matematika pre 5. ročník základných škôl*, 2. časť [Mathematics for the 5th grade, 2nd part]. SPN Bratislava.

DIGITAL INNOVATIONS IN MATHEMATICS TEACHER EDUCATION AT THE UNIVERSITY LEVEL

Julia Marie Stechemesser, Petra Scherer☜ and Florian Schacht

Abstract

For mathematics teacher education, the use of digital tools at the university level is of major importance, addressing both, content-related as well as educational-related topics. The workshop will present theoretical concepts and research findings from different projects. The participants will have the opportunity to explore and reflect selected digital learning materials, developed for different courses and modules.

Keywords: mathematics teacher education, digital tools, OER, interactive videos

The workshop focuses on the project DigiMal.nrw, which aims at developing

☜ University of Duisburg-Essen, Germany; e-mail: julia.stechemesser@uni-due.de; petra.scherer@uni-due.de