Designing a History and Philosophy of Mathematics Course for Preservice Teachers: An Emerging Framework

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DESIGNING A HISTORY AND PHILOSOPHY OF MATHEMATICS COURSE FOR PRESERVICE TEACHERS: AN EMERGING FRAMEWORK

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ABSTRACT

In this paper, we introduce an emerging framework for designing a History and Philosophy of Mathematics course at the master's level for preservice mathematics teachers (PMTs). The components are i) knowledge of mathematics and its history, ii) belief about/attitudes towards mathematics and its history, iii) beliefs about/attitudes towards using history in teaching mathematics and iv) design capacity of history-inspired mathematics lessons. We provide examples of certain topics that we used in our course and discuss how those contributed to our PMTs´ knowledge and beliefs connected to the above categories.

1 Introduction

Epistemology and historical developments of ideas, notions and mathematical objects have received particular attention from educational researchers. One can underline three main directions: *a*) the use of historical developments/ideas as a resource in teaching school mathematics (e.g., Ernest, 1998; Johansen & Kjeldsen, 2018); *b*) the use of historical developments/ideas as a tool and mediator for preservice and in-service teachers' and teacher educators' professional development (e.g., Jankvist et al., 2020; Turgut & Kohanová, 2021), and *c*) the interrelation between students' beliefs and historical developments of mathematical notions (e.g., Charalambous et al., 2009; Lada & Kohanová, in press). As a synthesis, the results underline the potential power of historical resources in teacher training and how this impacts their mathematical knowledge in teaching (Jankvist et al., 2020). In this paper, we report on ongoing research by focusing on b) and c) above and discuss an emerging framework (with its tentative four components) that can be used as a guide for designing a History and Philosophy of Mathematics (HPM) course for PMTs. Our particular aim is to explore dialectics through the use of history of mathematics, beliefs, and elaborating didactical designs as part of PMTs' professional development.

2 *Timeline and Background*

The framework introduced in this paper emerged from the research, which was initiated in 2018, under a pilot study entitled "Understanding student teachers' learning and development in the Historical and Philosophical Aspects of Mathematics course". The course is designed as a master-level course for PMTs with a two-fold aim. On the one hand, the aim is to provide information about historical, ontological and epistemological foundations for mathematical concepts and algorithms, as well as knowledge of what mathematics is, how its nature and methods developed, and what it constitutes today. On the other hand, it is aimed to develop PMTs' skills in transforming knowledge of the history of mathematics into didactical and pedagogical designs for teaching mathematics (grades 5-10). In the first years of the pilot study, we observed that PMTs had issues in designing lessons which incorporated the historical context meaningfully and that PMTs formed unexpected beliefs about mathematics and its development. This directed us toward systematic intentional inquiry to improve the practice and PMTs' outcomes, which is known as action research (Koshy et al., 2011). We have adopted an iterative approach embracing problem identification, action planning, implementation, evaluation, and reflection. The insights gained from the initial cycle fed into the planning of the second cycle, for which the action plan was modified, and the research process was repeated the next year (Figure 2).

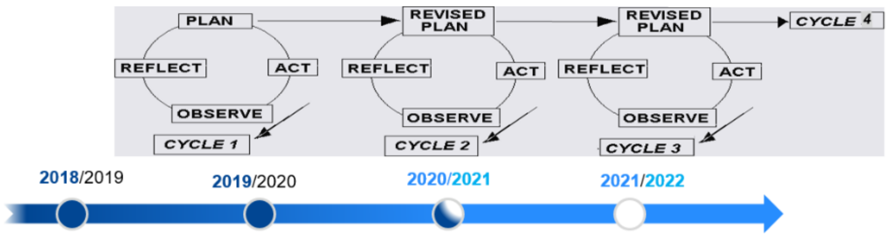

Figure 2. Iterative cycles applied in our action research

3 *The emerging framework*

As a first step for creating a framework for teaching history and philosophy of mathematics to PMTs, one needs to ask what the goals of such a course are. To answer this question, one needs to know what the goals of mathematics education for teachers are and how history of mathematics can contribute to achieving those. For the case of Norway, this has been discussed by Bjørn Smestad in the panel discussion of ESU6. Starting with two main goals of mathematics education for teachers: 1) to develop students' knowledge of mathematics and attitudes towards it, and 2) to develop their ability to teach mathematics, Smestad argued how history of mathematics can play a part in the whole spectrum of teacher knowledge (Barbin et al., 2011). The first of the above goals is a prerequisite for achieving the second one, which we view as the main goal of our teacher education program. Then, with 2) as a goal of the mathematics education for teachers, we set the main goal of the HPM course to be: *to develop the students' ability to teach mathematics using history of mathematics*. By "using history of mathematics" we mean that the students will become able to make choices informed by history of mathematics in planning, implementing and evaluating their lessons.

With the goal of the HPM set, the next question to ask is what a teacher needs to know to teach mathematics using history of mathematics. One needs *knowledge of mathematics and its history* which is the first component of our framework. By knowledge here we mean what Ball et al. (2008) call subject matter knowledge (SMK).

SMK alone is, of course, far from enough. If the PMTs are to use history of mathematics in their future work as mathematics teachers, they need to develop positive attitudes towards this approach. They need to believe that history has an important role to play in mathematics teaching and learning and that it is worth overcoming all the obstacles (Tzanakis & Arcavi, 2000) they might meet. *Beliefs about and attitudes towards using history in teaching mathematics* are therefore the second component of our framework.

The third component is PMTs' *beliefs and attitudes towards mathematics and its history*. It is often argued that history can be used to develop views on the nature of mathematics and mathematical activity (Tzanakis & Arcavi, 2000). But what type of views are cultivated within an HPM course? Special attention is needed, for example, to ensure that experimental and not formalist views on mathematics (Charalambous et al., 2009) will be strengthened.

Knowledge and beliefs/attitudes about mathematics and its history and about using history in mathematics do not automatically turn into lesson plans. Even if PMTs through an HPM course get inspired to use history in their future teaching, the course should also aim in equipping them with concrete methods for doing so. Thus, the fourth component of our framework is the PMTs' *design capacity of history-inspired mathematics lessons*.

To summarize, the emerging framework´s components are the domains of the PMT´s knowledge and beliefs/attitudes we believe are important for teaching mathematics using history of mathematics and which can be affected by an HPM course (Figure 1).

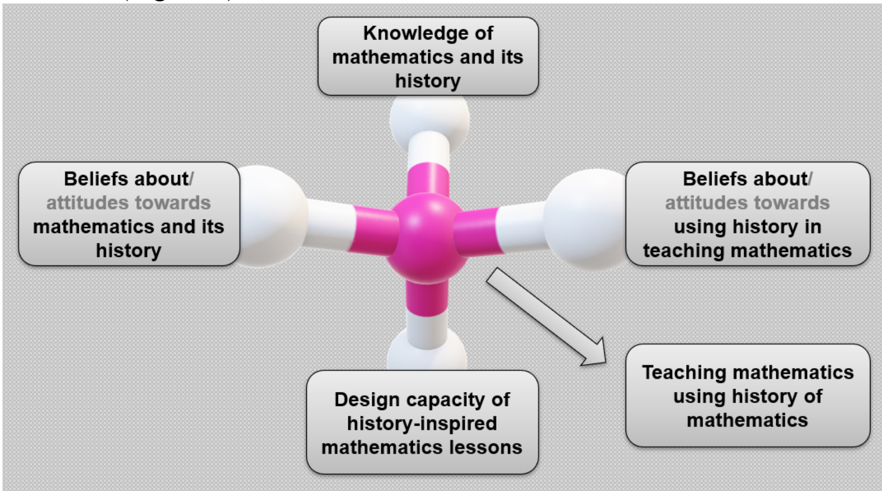

Figure 1. *The emerging framework*

The framework is theoretical (Fried, 2007) in the sense that at its core lies a set of questions. What type of knowledge, beliefs and attitudes do teachers need to use history in their teaching? How can an HPM course contribute to building those? As any theoretical framework though, it also has practical implications. The teacher educator can, by seeking to answer the questions above, use it as a guide for setting goals and choosing topics to include in an HPM course.

4 Examples

One of our intentions with including the three famous construction problems of Greek antiquity (quadrature of the circle, duplication of the cube, trisection of an angle), was that our PMTs discover the historical, social and cultural dimensions of mathematics (Hersh, 1994). In the session devoted to the topic, these three problems were presented to the PMTs and they worked hands-on with some of the efforts of Greek mathematicians of the time. The session ended with an outline of the mathematical advances that had to be made before an answer to the problem was given in the 19th century. In the written assignment that followed the PMTs were asked *what the construction problems teach us about mathematics and its development.* Analysis of their answers revealed that few PMTs grasped the sociocultural dimension of mathematics (Lada & Kohanová, in press) making evident that the expectation that certain beliefs will be formed just by exposing PMTs to history was unrealistic. In redesigning the course, we included discussions that brought to the surface the connection between mathematics and the cultural context it is born in.

Our second example is related to the lesson design component of our framework, and it is the product of a group of four mathematically strong PMTs' work within the workshop we organized in the spring semester of 2022. The group designed a lesson about Archimedes' Arbelos which was discussed in class in connection with the lunes of Hippocrates and the intention of showing how old mathematical problems can become the source of inspiration for newer ones. The PMTs created a task where GeoGebra was used to investigate the properties of the Arbelos. The only historical information included though was that "Archimedes found out this more than 2000 years ago". They were then challenged to reflect on how the historical aspect of their lesson supports pupils' learning. In the revised version they submitted, the PMTs added an introduction to their activity: "we start by asking the pupils how they thought a shoemaker's knife looked 2000 ago [and] ask them if they manage to see the knife [in the figure]". They claimed they used history as spices to increase pupils' motivation and that through their lesson "pupils can get an idea that mathematics is not universal truths but human-made conclusions", which is again doubtful considering that the problem was removed from its historical context.

5 *Concluding remarks*

Our framework emerged within a particular context. All choices, results and conclusions are tightly connected to the mathematical and pedagogical background of our PMTs, and the goals were set to fit the existing description of the HPM course and the expectations of our PMTs. We believe though it can potentially be used in designing an HPM course in different contexts. Last, as a next step, it would be interesting to follow our PMTs in practice and observe how they would manage to implement their lessons.

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