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ABSTRACT

The paper focuses on the skills of prospective mathematics teachers (PMTs) when planning lessons on primary and secondary school levels. The paper is based on a thematic analysis of PTMs' mathematics lesson plans developed within the frame of undergraduate courses at the Faculty of Education, Charles University in Prague, and Faculty of Mathematics, Physics and Informatics, Comenius University in Bratislava. Analysis of individual lesson plans, discussions with their authors, and comparison of the Czech and Slovak groups' lesson plans resulted in detecting and explaining the differences between the production of the two groups. The findings are of interest to teacher educators in general, researchers interested in teachers' lesson planning as well as practicing teachers.

KEYWORDS

A priori analysis, curricular spider's web, lesson planning, mathematics teacher education, thematic analysis

INTRODUCTION

The way how teachers reveal their knowledge rather than what they know is one of the reasons that makes mathematics teacher knowledge specialized (Scheiner et al., 2019). This specialized knowledge includes mathematical and pedagogical content knowledge as well as reflecting this knowledge on practice. Therefore, teachers of mathematics should master solving mathematical problems by selecting the most suitable problems and procedures for their pupils. Moreover, they should plan and present the lessons to stimulate pupils' cognitive processes. When discussing with teachers about their experiences from their professional life, they often mention the importance of having a good lesson plan. The form and level of details are not unified. It depends on many factors, e.g., the length of their teaching practice, self-confidence, and others. Some components can be found in nearly all lesson plans, others are based on the taught school subject or the teacher's personal preferences. For a successful starting of their teaching, PMTs need to be acquainted with the role and creation of lesson plans and their components.

The paper focuses on research in the domain of didactical engineering as defined and described in the Theory of Didactical Situations in Mathematics (TDSM) (Brousseau, 1997). The study enquires into lesson planning by PMTs at two universities, Charles University in Prague (Czech Republic) and Comenius University in Bratislava (Slovakia). The paper describes research designed and carried out to discover the key points that the participating PMTs consider as essential and include in their lesson plans. The well-prepared lesson plan contributes to the effective teaching and learning process.

The history of both countries, Slovakia and the Czech Republic, is connected in their past histories, even from the 833 AD. The educational systems in the current two independent countries develop separately, but they have many similarities. Several studies are documenting it in teacher education

(see, e.g., Slavičková and Novotná, 2022). On both faculties there are similar study programmes focusing on preparation of mathematics teachers. Differences are mostly in the emphasis put on the various components.

At both universities, the design of lesson plans is dealt with in the lessons of Didactics of mathematics at the Master's level of study. A deep analysis of lesson plans created by PMTs offers data for comparing the lesson plans, showing and explaining differences both locally and internationally.

Several studies are focusing on lesson plans from different perspectives. We present those that we used for making the list of the components used in this study. The list cannot be exhaustive, but it covers the main views on lesson planning. As our list of references documents, lesson plans are focused on in older as well as contemporary publications.

Brousseau (1997) presents a priori analysis as a crucial part of teachers' lesson planning. The theoretical background for a priori analysis is TDSM, and it is considered as one of the teachers' tools that they have when planning their lessons. Based on the lesson description, the teacher strives to estimate its course: uncover individual phases of the lesson, think about possible pupils' and the teacher's reactions (obstacles, errors, possibilities of preventing or correcting them), find out possible solving strategies for the problems that are planned to be solved (both correct as well incorrect ones) and preliminary knowledge needed for each strategy. In (Nováková and Novotná, 2011), real lesson plans of in-service teachers with a priori analyses according to TDSM are presented.

Rys (1975) categorizes lesson plans into three types. The 1st type answers the questions *What?* and *How?*. The 2nd type answers the questions *What was before?* and *What do I want to achieve?*. The 3rd type is the didactical analysis of the subject matter. The teacher works with aims describing what pupils should learn and to which extent, puts the didactical unit into the content and time structures (in the relationship with what was and what will be, see also Figure 1).

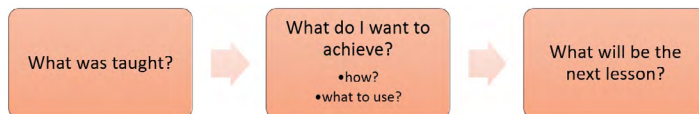


Figure 1: Teachers thinking about the lesson plan (according to Rys, 1975)

Rys classifies the questions into seven categories: questions about aims, content and teaching methods, specific didactical points of view, educational possibilities, organization of the teaching unit, timing, and realization of the lesson. Rys' view is near to that in (van den Akker, 2010), who presents the lesson plan components in the form of the so-called Curricular spider's web (Figure 2).

The Generalitat de Catalunya (2012)¹ considers as the common components for lesson planning content, teaching aims, learning outcomes, assessment, communication, cognitive skills and activities, resources, and procedure. The authors also list a set of reasons for planning a lesson, among which they incorporate the guidelines for planning, namely to: meet the students' needs and cater for different styles, control the time, assess students' performances, set targets, structure lesson-in, task, revision, plan for scaffolding, support the lower and higher cognitive skills according to Bloom's taxonomy².

1 This study focuses on lesson plans for CLIL (Content and Language Integrated Learning – teaching a non-linguistic subject in a language different from pupils' mother tongue), but it can be transferred to other lessons as well.

2 <https://evawintl.org/wp-content/uploads/Blooms-Taxonomy.pdf>

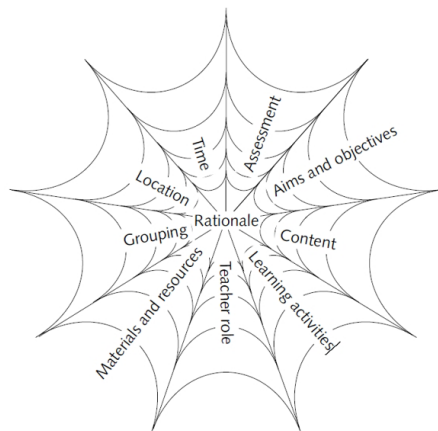


Figure 2: Van den Akker's (2010, p. 182) Curricular spider's web

The structure of the paper is as follows: the introduction is followed by the description of the methods for collecting and analysing data. Then the results are presented and discussed. After the discussion of results the conclusions are formulated.

In the paper, the following research questions are answered:

RQ1: What are the key components of lesson plans created by PMTs at the two universities?

RQ2: Do the Czech and Slovak PMTs' lesson plans components that they elaborate in their lesson plans differ, to what extent, and why?

MATERIALS AND METHODS

As mentioned in the Introduction, we worked with lesson plans from two groups of PMTs from Charles University, Faculty of Education ($N = 20$) and Comenius University, Faculty of Mathematics, Physics, and Informatics ($N = 19$). The analysis of the final versions of lesson plans was accompanied by discussions with the authors on the plans.

This research presents a content analysis of the 39 lesson plans. The approach can be characterized as thematic analysis. Braun and Clarke (2006) describe this method as “a method for identifying, analysing and reporting patterns (themes) within data” (p. 79). Based on literature and thematic analysis of PMTs' lesson plans, we created a list of key components.

We did not give participating PMTs a template to fill in. The preparation of their lesson plans was discussed in several lessons of didactics and didactics of mathematics. Therefore, some common schemes, but also significant differences among different groups of PMTs were identified in PMTs' lesson plans. When identifying key components, we did not consider those, which were not explicitly present in the lesson plan.

In both groups, PMTs could choose the topic for their lesson plans at their will. There was only one condition – it has to be focused on grades 5-13 (age 11-19). Looking at received lesson plans, we observed almost evenly distributed topics according to main school mathematics areas (Arithmetic; Algebra; Functions; Geometry and measurement; Combinatorics, probability, and statistic; and Reasoning and proof).

RESULTS

When categorizing PMTs' lesson plans, we used van den Akker's key components (2010) supplemented by three others (*Preliminary pupils' knowledge*, *Possible obstacles*, and *Summary*)

that were found frequently in PMTs lesson plans (Table 1). This helped us also to look at Rys’ “timeline” (Figure 1) and identify whether participating PMTs connected previous knowledge to the actual goal. Then we looked for connections (if any) to the future lesson(s).

Observing our data, we noticed that all PMTs included *Aims and Objectives*, *Content*, *Materials and Resources*, *Location* and *Time* in their lesson plans.

Aims and Objectives were formulated using the official state document (RVP in the Czech group and ŠVP in the Slovak group)³. Not all PMTs reformulated the official document sentences to fit the aim of their lesson plans.

Key characteristic	Czech Republic (max. 20)	Slovakia (max. 19)
Aims and Objectives	20	19
Content	20	19
Learning activities	7	19
Teacher role	20	3
Materials and Resources	20	19
Location	20	19
Time	20	19
Preliminary pupils’ knowledge	5	19
Possible obstacles	4	3
Assessment	11	10
Summary	5	8

Table 1: Key components and their occurrence in lesson plans

Material and Resources were identified in every analysed lesson plan. PMTs proposed different resources and sources. Even though textbooks were the primary source, some PMTs included e-sources, applications (e.g. GeoGebra, Kahoot!), instructional videos (mostly YouTube), and their own notes from secondary school.

Location and *Time* had almost the same structure in analysed lesson plans. *Location* was always in the classroom; *Time* was split into smaller slots for different activities in the hypothetical classroom. In some cases, the time estimation was not realistic, but PMTs have no (or small) teaching experiences, so it is a natural phenomenon.

In the other key components, there were visible differences in their occurrence in PMTs’ lesson plans. We focus on them in a more detailed way.

The category *Learning activities* was identified in 7 out of 20 Czech PMTs’ lesson plans, while all Slovak PMTs considered this characteristic as important. On the contrary, the category *Teacher role* was present only in 3 out of 19 Slovak PMTs’ lesson plans, but all Czech PMTs found it crucial to mention. These two points can be explained by the following reason: while the Czech group focused more on the teachers’ role and wrote the lesson plan focusing on him/her, the Slovak group found out the learning activities more important and omitted the teachers’ role in them. All 23 lesson plans contain creative learning activities, 6 PMTs considered also preparation for online teaching, not only offline. Manipulation with physical or virtual objects was identified in all lesson plans (e.g. introductory activity for making a hypothesis about the sum of inner angles in the triangle).

The other significant difference between the two groups is in considering *Preliminary pupils’ knowledge*. While in the Czech group, only 5 PMTs explicitly mentioned those, in the Slovak group, all PMTs did so. A possible reason could be the influence of the teacher educator. While in

3 RVP and ŠVP are systems of curricular documents for training students from 3 to 19 years. They define a binding framework for education. They are public documents available for teachers and non-teaching public.

the Slovak group, higher emphasis was put on the Rys' "timeline" (Figure 1) during the lessons, in the Czech group on Bloom's taxonomy. All Slovak PMTs tried to connect preliminary pupils' knowledge to the *Aims and Objectives* of the current lesson and to at least one following lesson. Moreover, one PMT made a connection to another topic area (his chosen topic was combinatorial numbers and he connected it also to probability). All Czech PMTs tried to assign tasks to different levels of Bloom's taxonomy.

We observed very close results in lesson plans in both groups concerning the categories *Possible obstacles*, *Assessment*, and *Summary*. Together, 7 PMTs (out of 39; 4 from the Czech group and 3 from the Slovak group) explicitly mentioned *Possible pupils' obstacles* during the lesson.

The unexpected phenomenon in both groups was the lack of *Assessment* in the PMTs' lesson plans. Explicitly stated assessment was in the lesson plans of 11 Czech PMTs and 10 Slovak PMTs. In several cases, the proposed assessment was vague (e.g. I'd ask them some questions), hypothetical (e.g. I'd give them a test), or completely missing. But there were individuals who suggested creative formative assessment or self-assessment of pupils (e.g., unfinished sentences like *I found out...*, *I learned...*, *I disliked...*).

Although TDSM (Brousseau, 1997) is dealt with in PMTs' didactical courses, the category *Summary* (corresponding with institutionalization in TDSM) was included in 13 lesson plans only. Among them, 5 Czech PMTs and 2 Slovak PMTs assigned institutionalization to the teacher, the others suggested making the summary by students themselves (e.g., exit tickets, class discussion, etc.)

DISCUSSION

The list of key components of lesson plans is the result of our analysis of those created by 39 PMTs. The participating PMTs had minimal teaching practice, therefore their lesson plans were based mainly on the knowledge gained during their teacher education courses. Their experience from their own school attendance may also contribute to differences in their lesson plan production. For teacher educators, it is of great importance to know what they can expect their students to know and to what it is important to pay greater attention.

When we compared our list of key components included in lesson plans with those published by other authors, we see that all of them have a common intersection but are not identical. The differences can have different reasons, e.g., differences in the organization of school systems, educational traditions, length of teachers' teaching practice, etc.

We found all the key components from our list in Brousseau's a priori analysis. Participating PMTs did not cover all Brousseau's categories, e.g., the analysis of obstacles and possible incorrect solving strategies were not considered in several participating PMTs' lesson plans. Nováková (2013) presents a detailed comparison of pre- and in-service teachers' lesson plans and a priori analysis and explains the detected differences.

We observed that PMTs tried to link the planned activities and the context of tasks to real-life. The inclusion of interdisciplinary connections was more frequent in the Slovak PMTs' lesson plans. We assign this difference to the fact that the Czech PMTs study resulted in obtaining the qualification as mathematics teachers, while the Slovak group has a two-subject study program (to become a teacher of mathematics in combination with another subject). It is obvious that this difference influenced the PMTs' template used for lesson plans.

In the presented results, participating PMTs did not adequately emphasize possible pupils' obstacles but they focused rather on the content and in most cases on the interaction with content. As Carrillo-Yañez et al. (2018) stated, understanding the nature of learning mathematics, which includes students' mathematical thinking, their interaction with content, and their strengths and weaknesses in learning the concept, is crucial for preparing lesson plans. Therefore, in our preparation of PMTs, more emphasis should be put on it.

Moraová and Novotná (2017) focused on PMTs' skills when planning CLIL lessons on primary and lower secondary school levels. Their attention was paid to the selection of topics the students find appropriate for a CLIL lesson, to the scope of activities they include, and to other components specific for CLIL lessons. In our groups of PMTs, the selection of the topics was given to their will. We identified more than half of the lesson plans focused on the introductory lessons.

To make a good lesson plan, a collaboration of teacher educators and PMTs is crucial. Stigler and Hiebert (1999) indicated four main phases of working on lesson plans: investigation, planning, enactment, and reflection. In our case, PMTs went through the first two phases. They set a learning goal and build a lesson plan. The created lesson plans were discussed with the peers and teacher educator. Savuran and Isikal-Bostan (2022) confirmed our observation from the presentation of lesson plans, that awareness of making the instructional decision can create PMTs' own path by being aware of the strengths of the pupils rather than applying the lesson plan as it is in the enacting phase.

CONCLUSION

In the presented paper we answered two research questions. Based on the literature review and thematic analysis we identified key components of the PMTs lesson plans. Based on their occurrence we compared two groups of PMTs and identified possible factors in differences in the lesson plans as described in detail in the text.

The analysis of the 39 PMTs' lesson plans showed that in general, PMTs are able to create utilizable lesson plans. Although making lesson plans is not the favourite activity of PMTs, it is an important part of their preparation for the career of mathematics teachers. It helps them realize what to pay attention to, what is important to focus on during teaching sequence, etc. When these lesson plans are discussed during the seminars, PMTs could see a different perspective and focus more on what is essential. If this part of PMTs' preparation is omitted, it could lead to a more difficult start of their full-time teaching practice.

The presented research pointed to the importance of doing thematic analysis of PMTs' lesson plans in order to finding the problematic parts, discussing them in a group, knowing how to start the lesson, what should be the next step, etc. A very similar idea was presented by Harmer (1992) who considered as one of the key questions for teachers' lesson planning the following: "What is it that my students will feel, know or be able to do at the end of the class (or classes) that they did not feel or know or were not able to do at the beginning of the class (classes)?" (p. 259).

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